

Mathematics 9062L, Analytic Number Theory Summer 2023, Western University

- **Instructor:** Masoud Khalkhali, Professor of Mathematics UWO.
Email: masoud@uwo.ca
Office: MC 137
Office hours: TBA, or by appointment.
- **Lectures:** MC 107, Tuesday and Thursday, 1-3 PM (with a short break). We start on Tuesday May 16 and will continue till July 1. To cover the missed first two weeks, we shall have extra hours in the first and third weeks of the course. This will be arranged in class.

Course outline: The following topics will be covered.

1) Primes in arithmetic progression (the theorem of Dirichlet): characters of finite abelian groups, analytic theory of Dirichlet series and L -functions, analytic continuation of the zeta function, Euler product formula, equi-distribution of primes in arithmetic progressions, a brief look at Chebotarev density theorem.

2) (Time permitting) Elliptic Modular forms: the modular group and its action on Poincare upper half plane- the modular curve, modular functions and modular forms, examples (Eisenstein series, elliptic functions, theta functions), the algebra of modular forms and its structure. Hecke operators, Hecke L -functions, applications to Ramanujan's conjectures on $\tau(n)$. Applications of theta functions to representation of integers by quadratic forms.

Suggested topics for student presentations:

1. Chebotarev density theorem
2. Prime number theorem
3. Selberg trace formula for Heisenberg group
4. Tate's thesis
5. Elliptic functions
6. Metaplectic representation and applications to Siegel modular forms
7. Four squares theorem of Lagrange via theta functions
8. Asymptotic evaluation of integrals (Steepest descent and stationary phase approximation methods, Airy function)
9. Two dimensional Galois representations
10. Riemann explicit formula and Riemann hypothesis

References:

1. The course textbook: A course in arithmetic; J. P. Serre (the last two chapters will be used).

The following are just for your own self study and won't be used in the lectures:

2. Automorphic Forms; A. Deitmar (for self study).
3. Introduction to analytic number theory; T. M. Apostol (for self study).
4. Euler through time; V. S. Varadarajan (for self study; mainly of historical interest. highly recommended!).
5. Automorphic forms and the Langlands program (for self study). Survey articles in this volume by Knapp, Gelbart, Gan, and Lapid are recommended.
6. The Riemann Hypothesis: Arithmetic and Geometry; Jeffrey C. Lagarias (in Surveys in Noncommutative Geometry)
7. Noncommutative Geometry and Number Theory; Paula Tretkoff (in Surveys in Noncommutative Geometry).

- **Important Remark:** Attendance and taking an active part in discussions in class is mandatory and very important for success in this course. Make sure not to miss any lectures, unless you have a valid documented reason.

Course evaluation: Your final mark will have three components: assignments %35 (3 sets of assignments will be given with 10 days due time for each), class attendance and active participation %15, your final essay and presentation %50. You must make sure to pick a topic for your essay in the first week and start preparing your latex and pdf file. No handwritten notes. I will discuss more details in the first lecture. You will find some potential topics in this course outline and in references cited above.

- **Conflict exams:** If you have a conflict with one of the exam times, please consult the Faculty of Science policy on missed course work. Based on that, if you think your situation qualifies you to take the conflict exam, please contact me as soon as possible, no later than a week before the exam in question.
- **Medical accommodations:** If you are unable to meet a course requirement due to illness or other serious circumstances, you must provide valid medical or other supporting documentation to the Dean's Office as soon as possible and contact me immediately. It is your responsibility to make alternative arrangements with me once the accommodation has been approved. In the event of a missed final exam, a "Recommendation of Special Examination" form must be obtained from the Dean's Office. For further information, please consult the University policy on medical accommodation.
- **Missed homework:** Late homework will not be accepted. Homeworks can always be submitted in advance. For extended absences or medical emergencies, these are handled the same way as for exams. In that case, a homework grade could be dropped; there will be no make-up homework.
- **Academic integrity:** Working on homework with your peers is allowed, in fact encouraged. However, each student must write their own solutions. Handing in suspiciously similar solutions will be considered an instance of cheating. Scholastic offences are taken seriously and will not be tolerated. For more information, please consult the University policy on scholastic discipline.

- **Accessibility:** Please consult Services for Students with Disabilities (SSD) regarding accessibility services on campus. Please contact me if you require material in an alternate format or other accommodations to make this course more accessible to you.